LDA equations

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\begin{aligned}
&p_1(x_0) > p_2(x_0)\\
&\Rightarrow \frac{\pi_1 \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x_0-\mu_1)^2\right)}{\sum_{l=1}^{K}\pi_l \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)} > \frac{\pi_2 \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x_0-\mu_2)^2\right)}{\sum_{l=1}^{K}\pi_l \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)} \quad \textit{common denom}\\
&\Rightarrow \pi_1 \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x_0-\mu_1)^2\right) > \pi_2 \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2\sigma^2}(x_0-\mu_2)^2\right) \quad \textit{rm constant, then log}\\
&\Rightarrow \log(\pi_1) - \frac{1}{2\sigma^2}(x_0-\mu_1)^2 > \log(\pi_2) - \frac{1}{2\sigma^2}(x_0-\mu_2)^2 \quad \textit{shift sides, expand}\\
&\Rightarrow -(x_0^2 - 2x_0\mu_1 + \mu_1^2) + (x_0^2 - 2x_0\mu_2 + \mu_2^2) > 2\sigma^2(\log(\pi_1) - \log(\pi_2)) \quad \textit{cancel and simplify}\\
&\Rightarrow 2(\mu_1-\mu_2)x_0 - (\mu_1^2 - \mu_2^2) > 2\sigma^2(\log(\pi_2) - \log(\pi_1)) \quad \textit{expand}\\
&\Rightarrow 2(\mu_1-\mu_2)x_0 - (\mu_1-\mu_2)(\mu_1+\mu_2) > 2\sigma^2(\log(\pi_2) - \log(\pi_1)) \quad \textit{simplify}\\
&\Rightarrow 2x_0 - (\mu_1+\mu_2) > 2\sigma^2\frac{\log(\pi_2)-\log(\pi_1)}{(\mu_1-\mu_2)} \quad \textit{shift sides}\\
&\Rightarrow x_0 > \frac{(\mu_1+\mu_2)}{2} + \sigma^2\frac{\log(\pi_2)-\log(\pi_1)}{(\mu_1-\mu_2)})
\end{aligned}
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if $\pi_1 = \pi_2$ then $x_0 > \frac{\mu_1+\mu_2}{2}$